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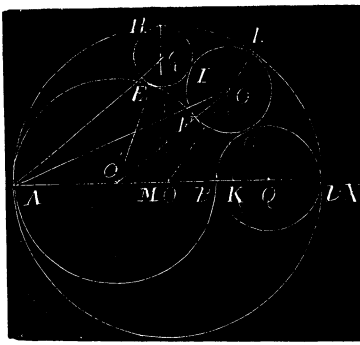
ANSWER TO QUERY AT PAGE 143, *VOL. V.*

BY GEORGE EASTWOOD, SAXONVILLE, MASS.

Analysis.—Suppose the costruction effected, and that the required circles O, O', O'', are drawn so as to touch each other and the given circles Q and Q', as in the annexed diagram.



Join the centers O and O''; O' and O''; O and Q'; and O' and Q'. Produce OO'' to L, and OQ' to H, and upon AD demit the perpendiculars $O''K$ and $Q'M$.

By a propety of the Arbelos (See Leybourn's Math. Rep., pp. 207–209), $O''K = 2FO''$, $Q'M = 4EQ'$, etc. [See ANALYST, p. 171, Vol. I.]

From the triangle $O'O''O$, we have

$$(O'F+FO'')^2 = O'O^2+O''O^2+2O'O.OK$$
$$=(AO-AO')^2+(OL-O'L)^2+2(AO-AO')\,OK$$
$$=(AO-AO')^2+(AO-FO'')^2+2(AO-AO')\,OK;$$

whence $$OK = \frac{(AO+AO')FO''}{AO-AO'} - AO.$$

Make $AO = R$, $AO' = R'$, $BQ = R''$, $FO'' = r$, and $EQ' = r'$; then, since $R = R'+R''$, we have

$$R+R' = 2R'+R'',\ R-R' = R'',$$

and therefore $$OK = \frac{(2R'+R'')r}{R'} - (R'+R''). \qquad (A)$$

But $OK^2 = OO''^2 - O''K^2 = OO''^2 - (2FO'')^2 = (R'+R''-r)^2 - 4r^2$. (B)

Putting the square of (A) in (B), we deduce

$$r = \frac{(R'+R'')R'R''}{R''^2+(R'+R'')R'}. \qquad (C)$$

By a similar process we deduce

$$r' = \frac{(R'+R'')R'R''}{4R''^2+(R'+R'')\,R'}. \qquad (D)$$

By the question, $r' = EQ' = Q'H$, is a given line. Therefore, solving the symbolical equation (D) for R', we readily find

$$R' = \tfrac{1}{2}R''[\sqrt{(R''+15r')}-1],$$
$$R = \tfrac{1}{2}R''[\sqrt{(R''+15r')}+1],$$
$$r = \frac{R''(R''+15r'-1)}{R''+15r'+3}.$$

Furthermore, adding AO to OK we find

$$AK=\frac{(AO+AO')FO''}{AO-AO'}=\frac{(2R'+R'')r}{R''}=\frac{R''(R''+15r')^{\frac{1}{2}}(R''+15r'-1)}{R''+15r'+3},$$

and, similarly,

$$AM=\frac{(AO+AO')EQ'}{AO-AO'}=\frac{(2R'+R'')r'}{R''}=r'\sqrt{(R''+15r')}.$$ Hence this

Construction.—On the indefinite straight line AX, set off

$$AB=2AO'=\tfrac{1}{2}R''[\sqrt{(R''+15r')}-1],$$
$$AD=2AO=\tfrac{1}{2}R''[\sqrt{(R''+15r')}+1],$$
$$BQ=R''.$$

Bisect AB in O', and AD in O: then, with centers O', O and Q, and with radii AO' AO and BQ, describe the circles AEB, $AHLD$ and BGD. These will obviously touch each other, two and two.

Again, on AD set off

$$AK=\frac{R''\sqrt{(R''+15r')}(R''+15r'-1)}{R''+15r'+3},$$

and $AM=r'\sqrt{(R''+15r')}$. At K, erect the perpend. KO''

$$=\frac{2R''(R''+15r'-1)}{R''+15r'+3},$$

and at M erect the perpendicular $MQ'=4r'$. Finally, with centers O'' and Q', and radii $FO''=R''(R''+15r'-1)\div(R''+15r'+3)$ and $EQ'=4r'$, describe the circles FIL, EIH and it is done.

[The foregoing solution, it will be seen, applies only to a special case of the question quoted at p. 143, Vol. V, as the relative position of the given circles cannot be arbitrarily taken. No general solution has been rec'd.]

NOTE ON THE DIFFERENTIATION OF EXPONENTIAL AND TRIGONOMETRICAL FUNCTIONS.

BY ARTEMAS MARTIN, M. A., ERIE, PA.

THE following methods may not be new, but I have not met with them in any of the many works on the calculus to which I have access.

1. Differentiate $y=a^x$.

We have, by the Exponential Theorem,

$$y=a^x=1+x\log a+\frac{x^2(\log a)^2}{1.2}+\frac{x^3(\log a)^3}{1.2.3}+\text{ etc};$$